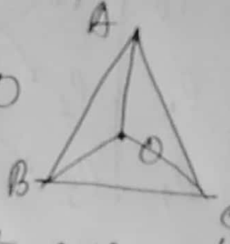
Symmetrics of an equilateral trinangle.

Let S be the set of all points in a Euclidean space. An isometry of the space is a bijection of S onto S that preserves distance between two points in S.

A symmetry of a geometrical figure in a Euclidean space is an isometry that keeps the figure as a whole unchanged.

For example, let T be an equilateral triangle with its centroid O in a plane π and ρ be a rotation in the plane about O through 120°. Then ρ effects permutation of the ~~ver~~ vertices but maps the triangle as a whole onto itself. ρ is a symmetry of the triangle T.

Let ABC be an equilateral triangle with centroid O. There are six symmetries of the triangle.



i = rotation in the plane about O through ~~120~~ 0°

r_1 = rotation in the plane about O through 120°

r_2 = rotation in the plane about O through 240°

a = reflection about AO

b = reflection about BO

c = reflection about CO

Let 'o' stands for the composition of mappings. Then $r_1 \circ r_2 = r_2$; $a \circ b = r_1$; $b \circ a = r_2$ etc.

Let $S = \{i, r_1, r_2, a, b, c\}$. Taking 'o' as the binary composition on S, the composition table is given below.

$\circ$	i	r_1	r_2	a	b	c
i	i	r_1	r_2	a	b	c
r_1	r_1	r_2	i	c	a	b
r_2	r_2	i	r_1	b	c	a
a	a	b	c	i	r_1	r_2
b	b	c	a	r_2	i	r_1
c	c	a	b	r_1	r_2	i

— 1

Note:- The three rotations correspond to the following permutations of the vertices -

$$i = \begin{pmatrix} A & B & C \\ A & B & C \end{pmatrix}; \quad r_1 = \begin{pmatrix} A & B & C \\ B & C & A \end{pmatrix}$$

$$r_2 = \begin{pmatrix} A & B & C \\ C & A & B \end{pmatrix};$$

The reflections correspond to the following permutations of the vertices -

$$a = \begin{pmatrix} A & B & C \\ A & C & B \end{pmatrix}; \quad b = \begin{pmatrix} A & B & C \\ C & B & A \end{pmatrix}$$

$$c = \begin{pmatrix} A & B & C \\ B & A & C \end{pmatrix}$$

The dihedral group D_3 is same as the symmetric group S_3.

Dihedral group :- clearly from the composition table (1) the six symmetries of the ~~equa~~ equilateral triangle form a non commutative group. This group is called the dihedral group and is denoted by D_3.